# Quaternion Rotations

A quaternion is a number in the form of

where x, y, z, and t are real numbers. **i**, **j**, and **k** are square roots of -1, and, **ijk =** -1

A pure quaternion is a number in the form of

where x, y, and z are real numbers.

The modulus of a quaternion is

given that **q** is a quaternion defined as before.

A unit quaternion is a pure quaternion whose modulus is 1

Given quaternion **q** the conjugate of **q** is defined as

**\* =**

To add or subtract quaternions, separately add or subtract the coefficients of **i**, **j**, and **k** and add the real numbers t together

The product of a quaternion and a scalar is defined as

The Dot Product of two pure quaternions is defined as

Where 𝝓 is the angle between the quaternions in Euclidean three-dimensional space

The Cross Product of two pure quaternions is defined as

Where 𝝓 is the angle between the quaternions in Euclidean three-dimensional space and **n** is a pure unit quaternion representing a unit vector in three-dimensional space orthogonal to the plane of the two three-dimensional vectors represented by quaternions and .

The multiplication of two pure quaternions is defined as

The polar representation of a quaternion is

where is a pure, unit quaternion

Given pure quaternion and unit quaternion . Let be the transformation defined by

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is the rotation of the point in three-dimensional space represented by about the axis represented by by the angle .

Because

Since will always be orthogonal to .

Because,

By Lagrange’s Triple Product Formula

And Because,

By the trig identities

# Quaternion Rotation Combinations

Let there be two Quaternion Rotations,

Dividing the equality by the law of cosines on a sphere

Results in

Multiplying the left side by

From this we can find that

By the law of cosines on a sphere and because

And

And

There are 2 cases where this is undefined, however, those two cases are when

And these rotations both result in no rotation